Sensible and Latent Heat Flux Model Updates

Wave Augmented Scalar Flux

- Bulk Formula

$$\frac{\tau}{\rho} = u_*^2 = C_D \Delta U^2 \ , \ \frac{H}{\rho c_p} = \theta_* |u_*| = C_H \Delta\theta |\Delta U| \ , \text{ and } \frac{Q}{\rho} = q_* |u_*| = C_Q \Delta q |\Delta U|$$

- Sea State Augmented Scalar Flux Profiles:

$$\begin{aligned}
\Delta U &= \left[1 - \exp\left(\frac{-z^+}{Z_A}\right)\right] \left[\frac{Z_A u_{*_\nu} |u_{*_\nu}|}{|u_*|} + \frac{u_*}{\kappa}\left\{\ln\left(\frac{z+\delta_o}{\delta_o}\right) - \Psi_o\left(\frac{z}{L}\right)\right\}\right] , \\
\Delta\theta &= \left[1 - \exp\left(\frac{-z^+ Pr}{Z_B}\right)\right] \left[\frac{Z_B \theta_{*_{mol}} |u_{*_\nu}|}{|u_*|} + \frac{\theta_*}{\kappa}\left\{\ln\left(\frac{z+\delta_\theta}{\delta_\theta}\right) - \Psi_\theta\left(\frac{z}{L}\right)\right\}\right] \\
\Delta q &= \left[1 - \exp\left(\frac{-z^+ Sc_q}{Z_C}\right)\right] \left[\frac{Z_C q_{*_{mol}} |u_{*_\nu}|}{|u_*|} + \frac{q_*}{\kappa}\left\{\ln\left(\frac{z+\delta_q}{\delta_q}\right) - \Psi_q\left(\frac{z}{L}\right)\right\}\right]
\end{aligned}$$

Wave Augmented Scalar Flux

- Molecular Flux Component:

$$\left.\begin{aligned} u_{*_\nu} &= \sqrt{f(u_{*_s})^2} \\ \theta_{*_{mol}} &= \sqrt{f\theta_{*_s}^2} \\ q_{*_{mol}} &= \sqrt{f q_{*_s}^2} \end{aligned}\right\} \quad \begin{aligned} H_{mol} &= c_p \rho \theta_{*_{mol}} |u_{*_\nu}| \\ Q_{mol} &= \rho q_{*_{mol}} |u_{*_\nu}| \end{aligned}$$

- Non-Molecular Flux Component:

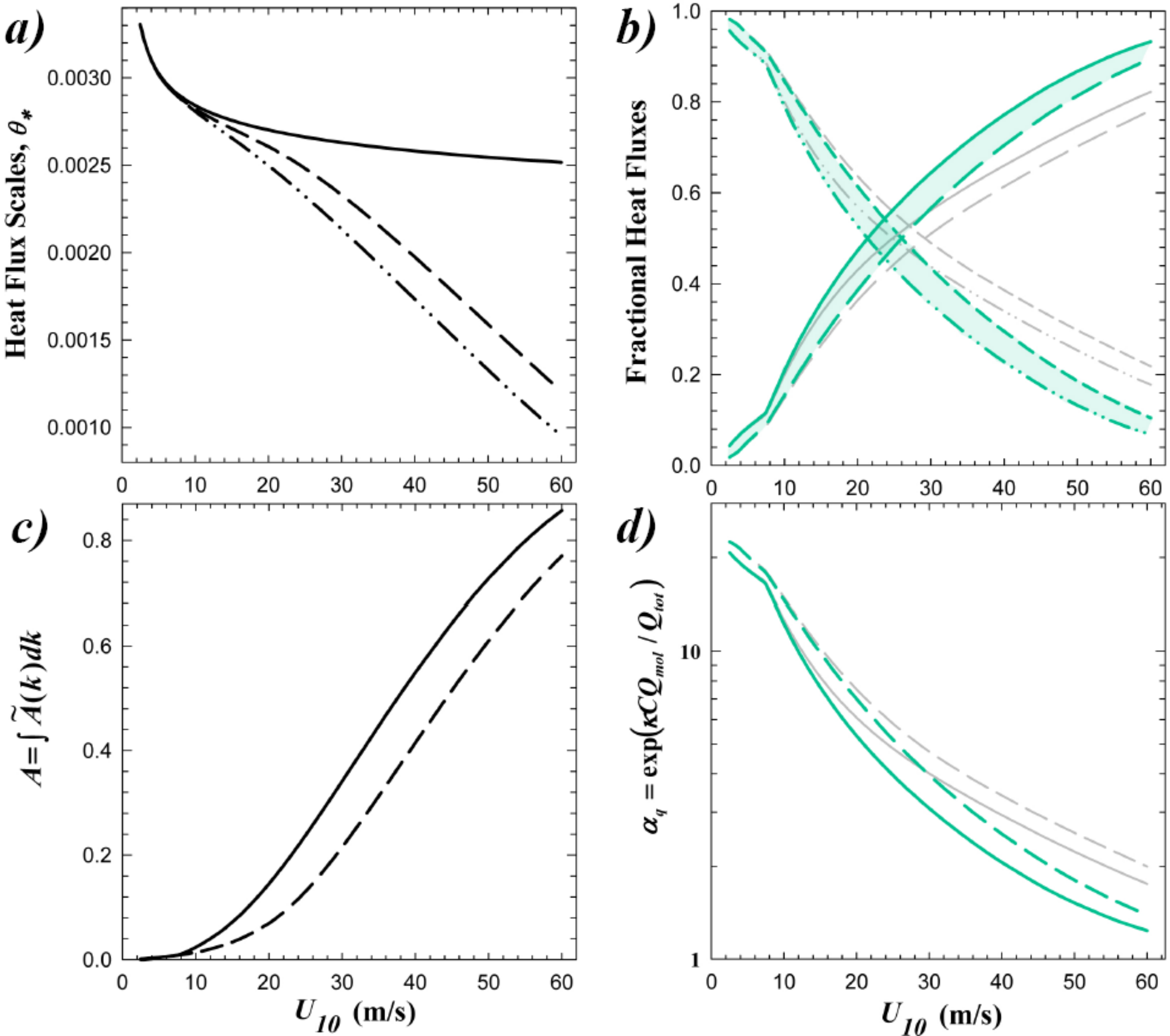
$$H_{non} = H_{tot} - H_{mol}$$

$$Q_{non} = Q_{tot} - Q_{mol}$$

$$\boxed{\theta_{*_{non}} = \frac{\theta_* |u_*| - \theta_{*_{mol}} |u_{*_\nu}|}{|u_{*_{turb}}|}}$$

$$\boxed{q_{*_{non}} = \frac{q_* |u_*| - q_{*_{mol}} |u_{*_\nu}|}{|u_{*_{turb}}|}}$$

Wave Augmented Scalar Flux



Note: This does not include work on enthalpy fluxes

Current Gas Flux Model

Gas Flux Model Concept

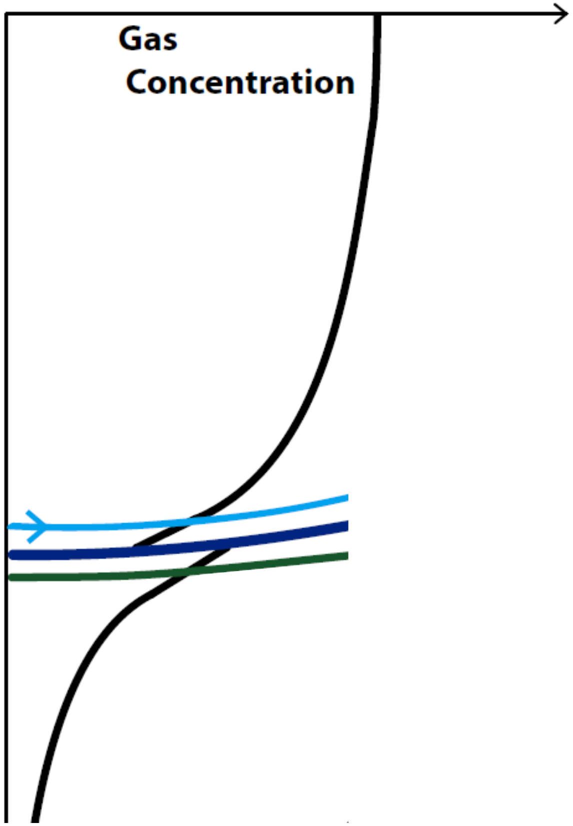
$$Flux = k(\chi_{water} - \alpha\chi_{air})$$

<u>Oceanic</u>:

- Temperature
- Salinity
- Currents
- Gas Concentrations

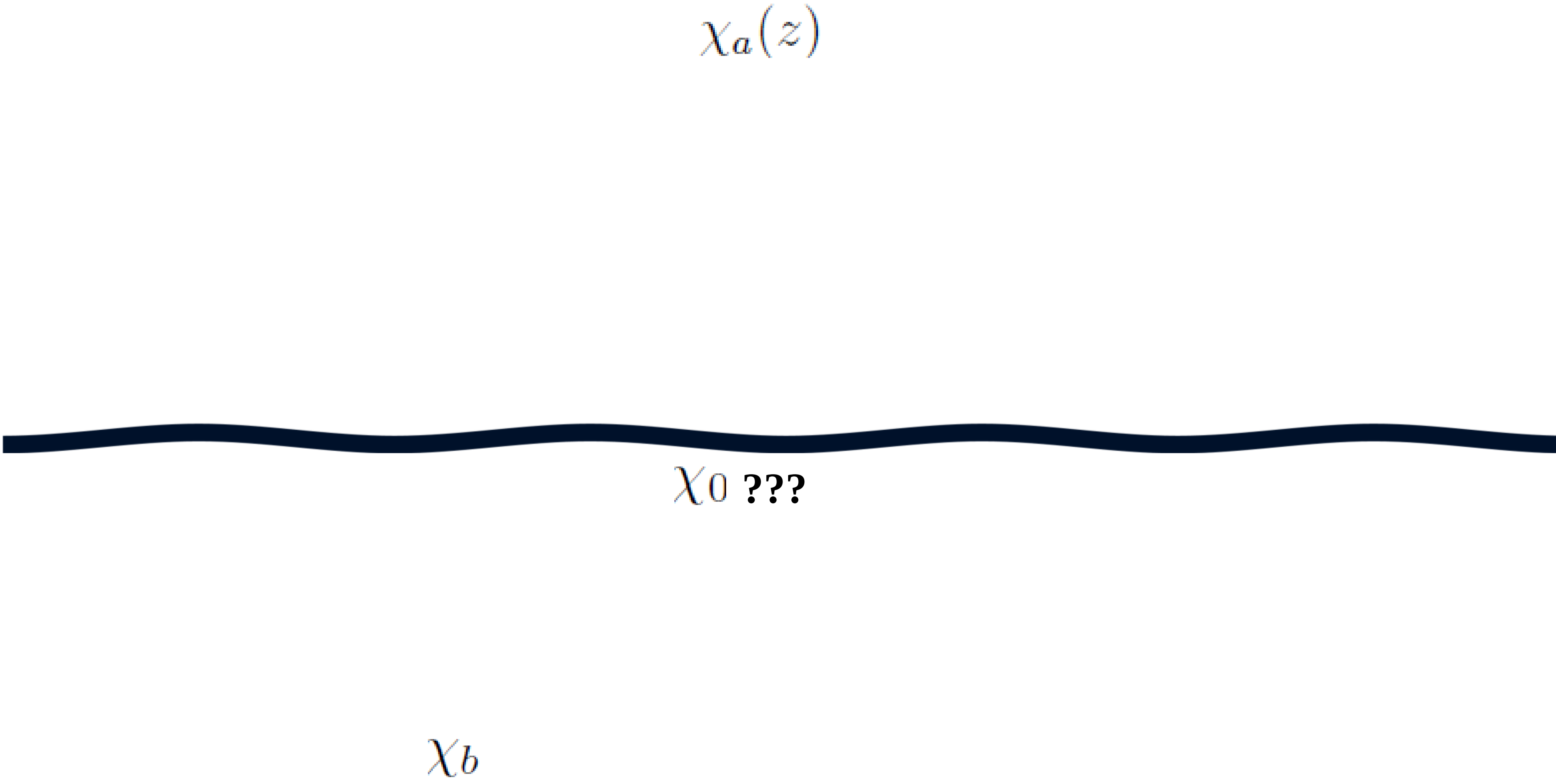
<u>Meteorological:</u>

- Wind Speed
- Pot. Temperature
- Specific Humidity
- Gas Concentrations



How the Interfacial Model Works

- Initial Conditions:



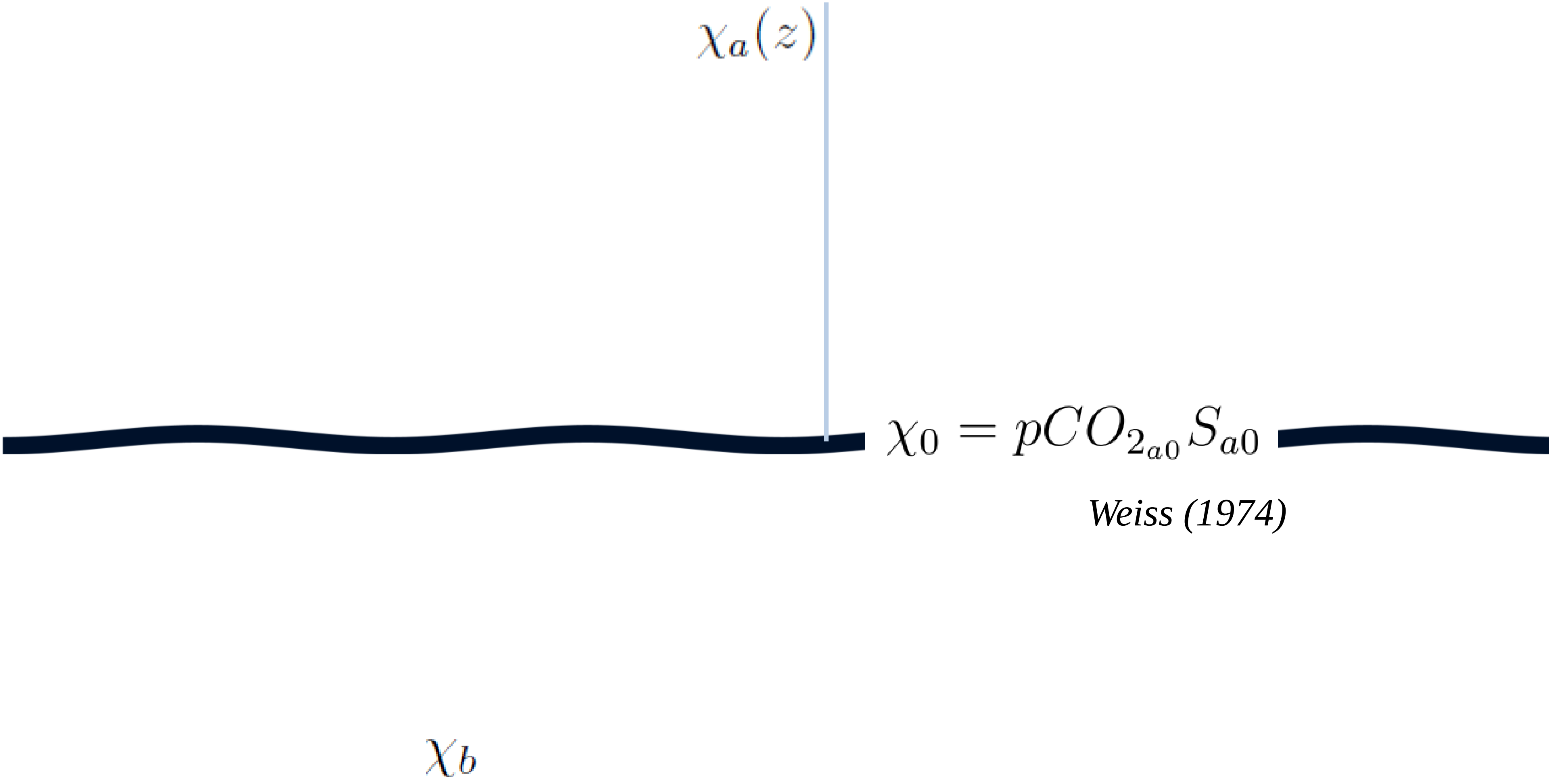
$\chi_a(z)$

χ_0 **???**

χ_b

How the Interfacial Model Works

- Initial Conditions:



$\chi_a(z)$

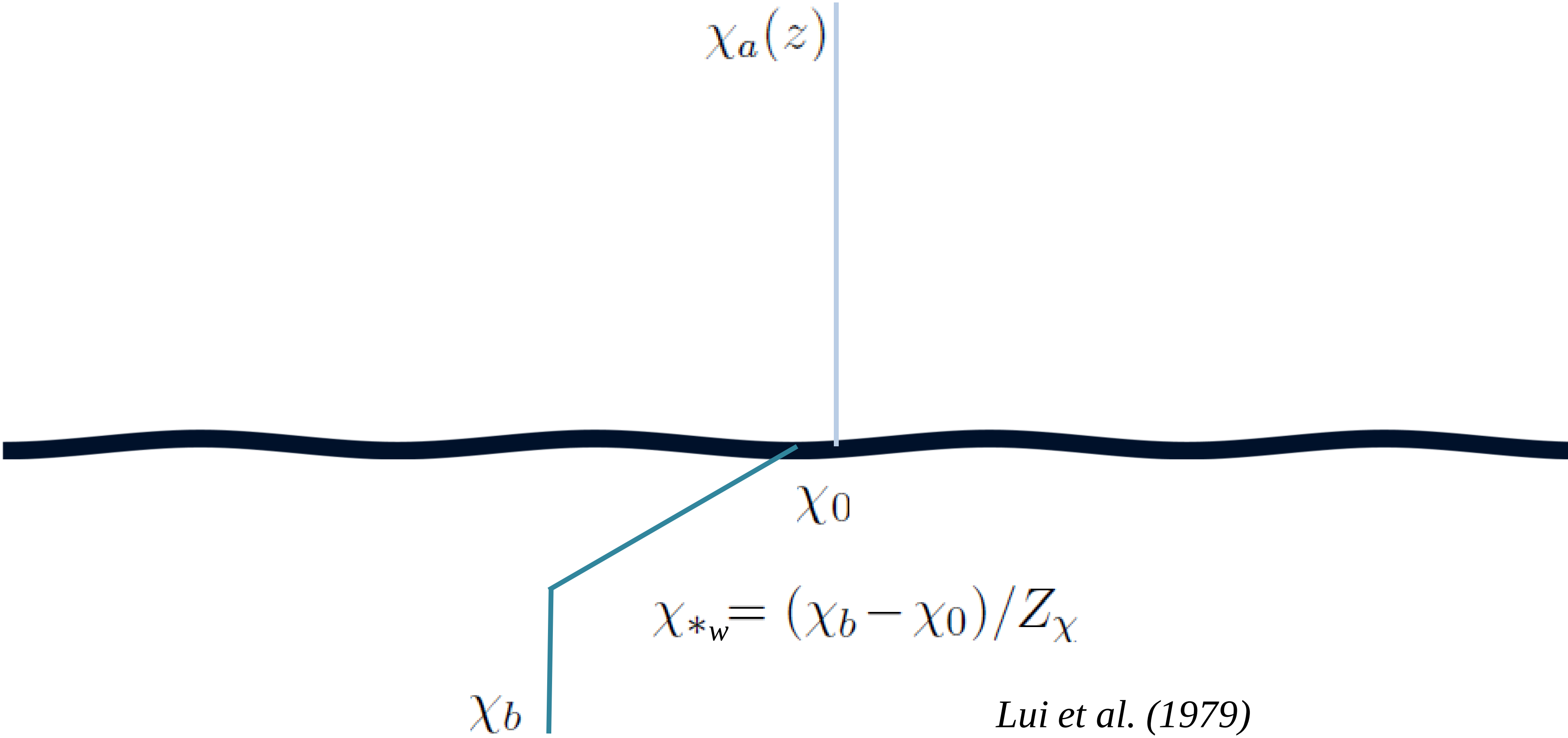
$$\chi_0 = pCO_{2_{a0}} S_{a0}$$

Weiss (1974)

χ_b

How the Interfacial Model Works

- Initial Flux:

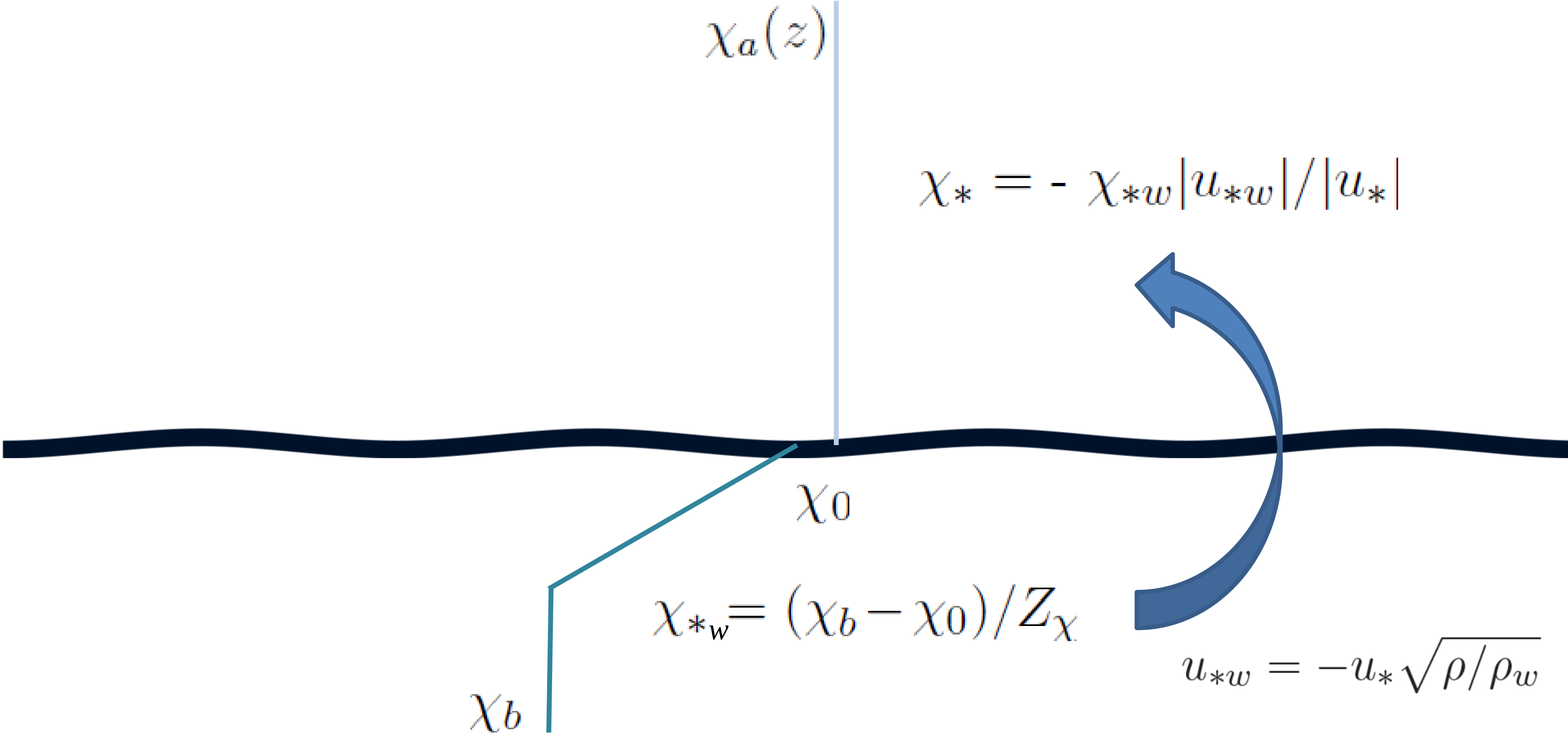


$$\chi_{*w} = (\chi_b - \chi_0)/Z_\chi$$

Lui et al. (1979)

How the Interfacial Model Works

- Flip Flux Scale to Atmosphere:



$$\chi_* = - \chi_{*w}|u_{*w}|/|u_*|$$

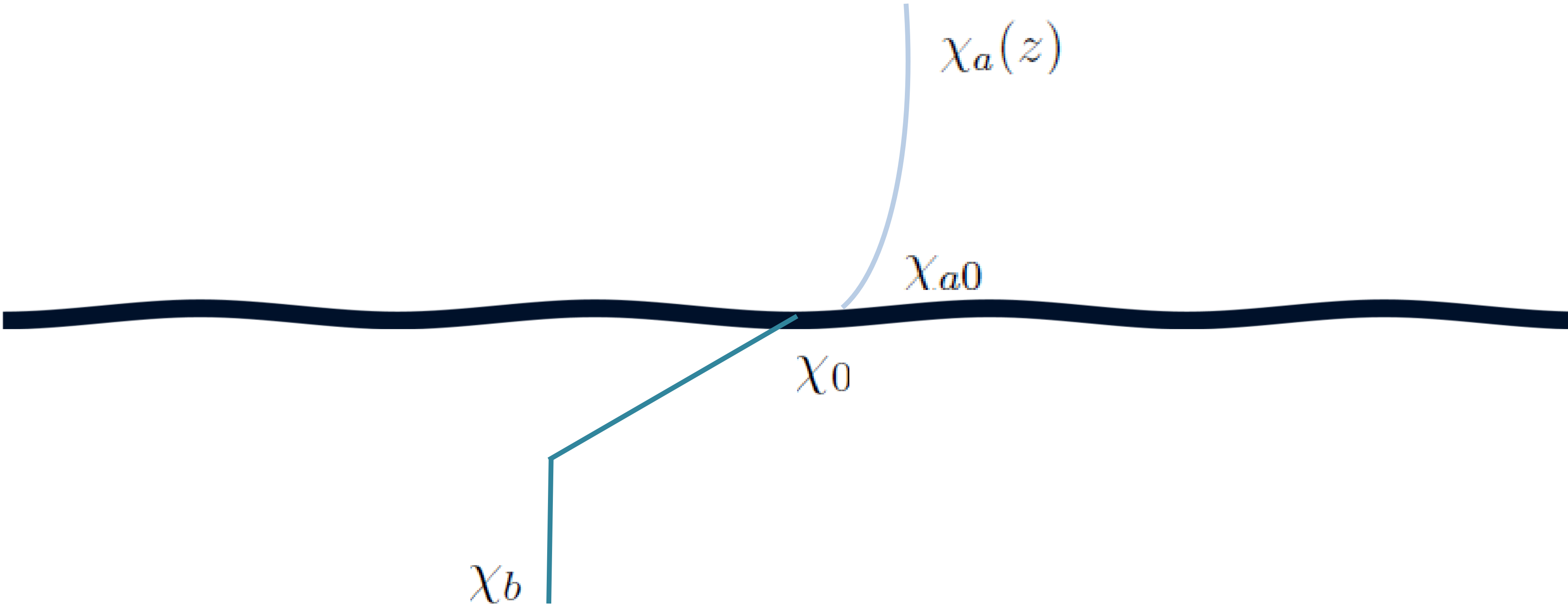
$$\chi_{*w} = (\chi_b - \chi_0)/Z_\chi$$

$$u_{*w} = -u_*\sqrt{\rho/\rho_w}$$

How the Interfacial Model Works

- Atmospheric Profile:

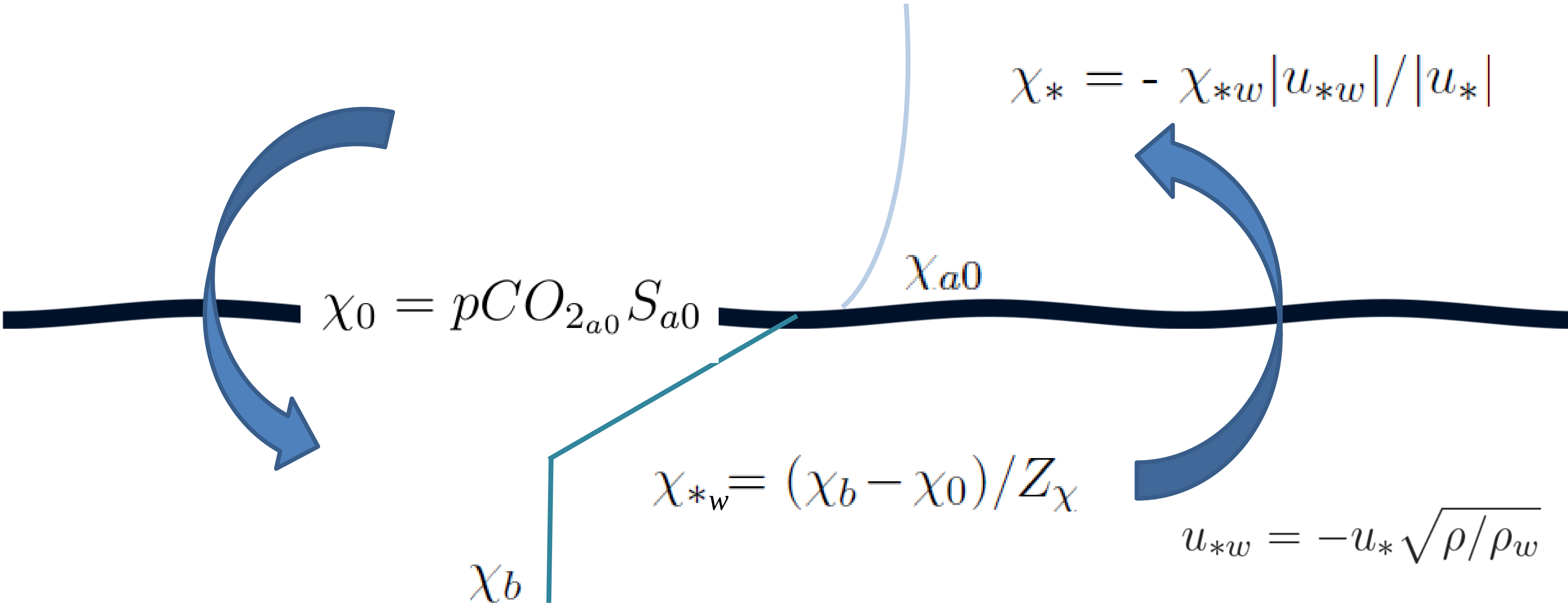
$$\frac{\chi_a(z)-\chi_{a0}}{\chi_{a*}} = E_a\left[1-\exp\left(\frac{-z}{\epsilon_{\chi_a}}\right)\right]\left|\frac{u_{*_\nu}}{u_*}\right| + \frac{1}{\kappa}\left[\ln\left(\frac{z+\delta_{\chi_a}}{\delta_{\chi_a}}\right)-\Psi_{\chi_a}\right]\left[1-\exp\left(\frac{-z}{\epsilon_{\chi_a}}\right)\right]$$



How the Interfacial Model Works

- Iterate:

$$\frac{\chi_a(z)-\chi_{a0}}{\chi_{a*}} = E_a\left[1-\exp\left(\frac{-z}{\epsilon_{\chi_a}}\right)\right]\left|\frac{u_{*_\nu}}{u_*}\right| + \frac{1}{\kappa}\left[\ln\left(\frac{z+\delta_{\chi_a}}{\delta_{\chi_a}}\right)-\Psi_{\chi_a}\right]\left[1-\exp\left(\frac{-z}{\epsilon_{\chi_a}}\right)\right]$$



$$\chi_* = - \chi_{*w}|u_{*w}|/|u_*|$$

χ_{a0}

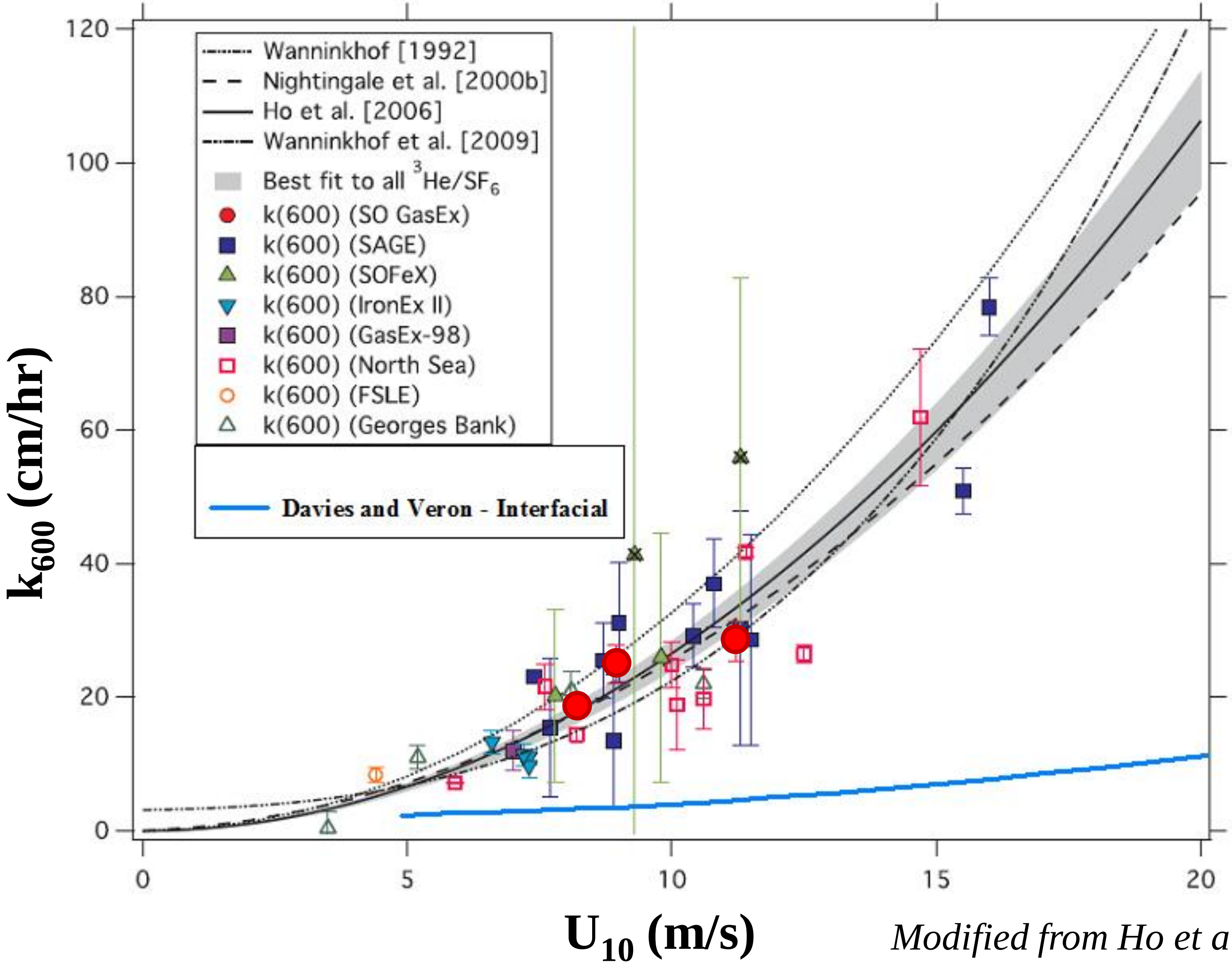
$$\chi_0 = pCO_{2_{a0}} S_{a0}$$

$$\chi_{*w} = (\chi_b - \chi_0)/Z_\chi$$

$$u_{*w} = -u_* \sqrt{\rho/\rho_w}$$

χ_b

Interfacial Flux Model Results



Modified from Ho et al. (2011)

Improving the Flux Model

$$Flux = k(\chi_{water} - \alpha\chi_{air})$$

Oceanic:

- Temperature
- Salinity
- Currents
- Gas Concentrations

Meteorological:

- Wind Speed
- Pot. Temperature
- Specific Humidity
- Gas Concentrations

Sea State:

- Fetch
- Wind Speed

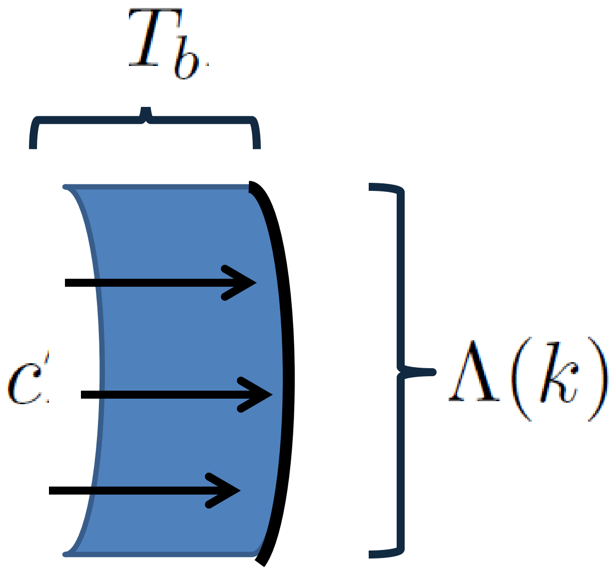
Bubble Mediate Transfer

- From Woolf, 1997:

$$k_b = \frac{YW_c}{\omega}\left[1+\left(e\omega S_{c\chi w}^{1/2}\right)^{-1/n}\right]^{-n}$$

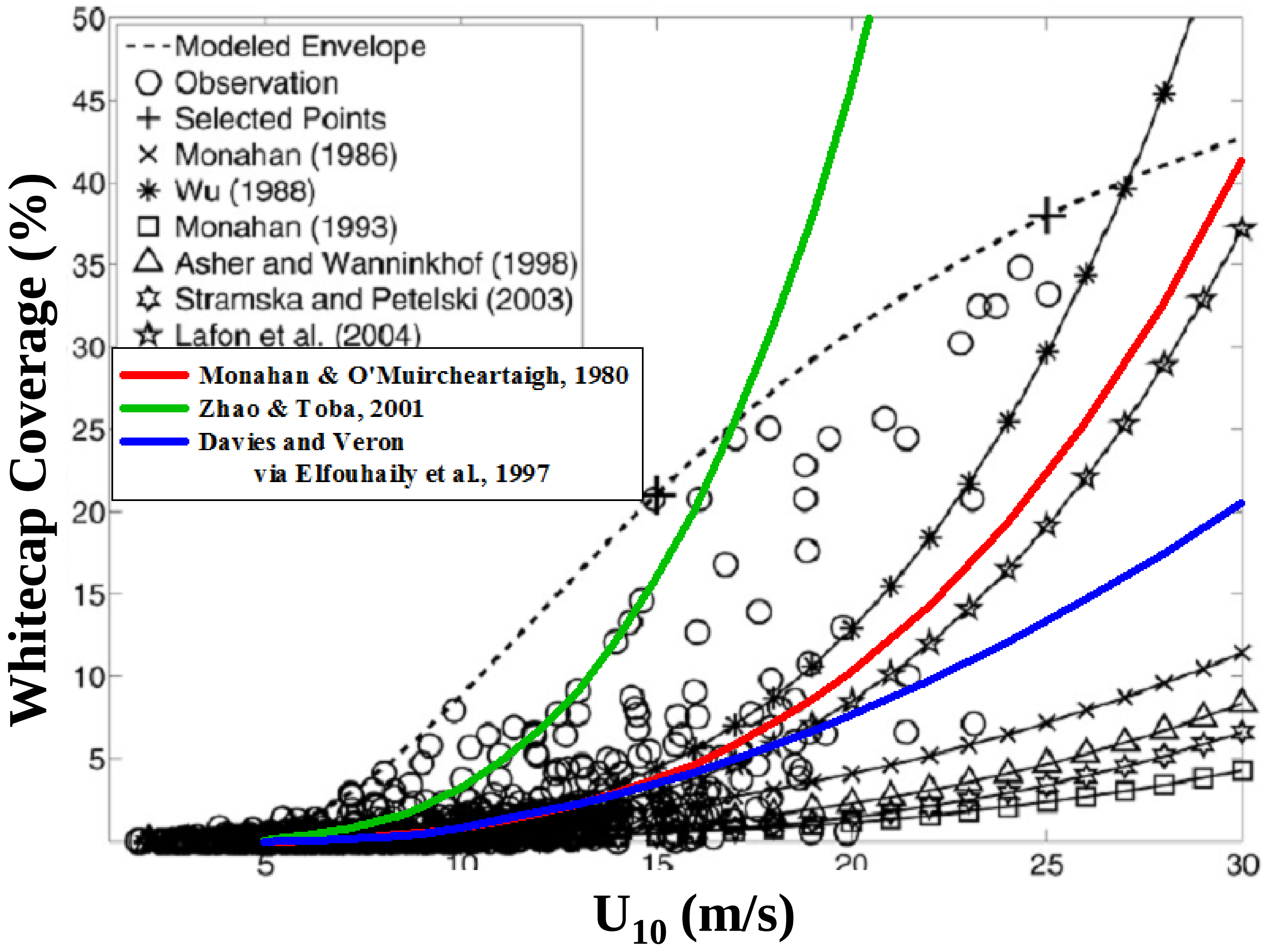
- Whitecap Coverage:

$$W_c = \int_k cT_b\Lambda(k)dk$$



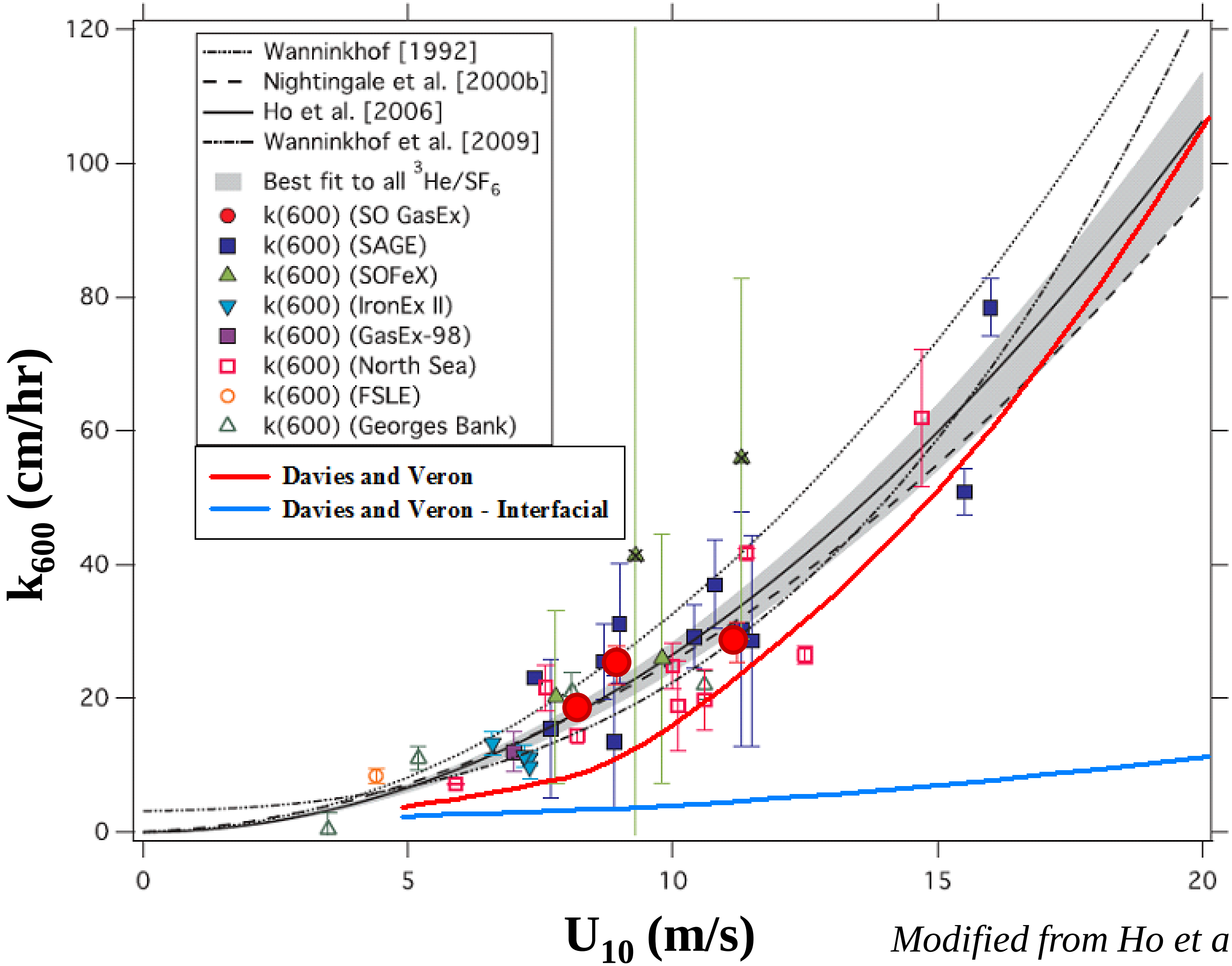
See: Phillips (1984)

Modeled Whitecap Coverage



Modified from Anguelova and Webster (2006)

Flux Model Results



Modified from Ho et al. (2011)

Proposed Gas Flux Model

Energy Dissipation

- Phillips 1984 gave the that the total energy dissipation rate as:

$$\varepsilon(c)dc = \frac{\rho b c^4}{g} c\Lambda(c)dc \sim \frac{\left[\frac{kg}{m^3}\right][\ ]\left[\frac{m^4}{s^4}\right]}{\frac{m}{s^2}} \left[\frac{m}{s}\right]\left[\frac{1}{m}\right] = \left[\frac{kg}{s^3}\right] \sim \left[\frac{kgm^2}{m^2 s^3}\right] = \left[\frac{J}{sm^2}\right]$$

- Multiplying by the breaking time:

$$E(c)dc = T\varepsilon(c)dc = \frac{\rho b c^4}{g} Tc\Lambda(c)dc \sim \left[\frac{J}{sm^2}\right][s] = \left[\frac{J}{m^2}\right]$$

Length of Break Wave

- Length of breaking wave crest per unit area between the intervals *k* to *k+dk:*

$$\Lambda(k,\theta)k\,d\theta\,dk = \frac{\beta(k,\theta)k^4\Psi(k,\theta)}{\omega b}d\theta\,dk$$

$$\int_k \Lambda(k)dk = \int_k \int_\theta \Lambda(k,\theta)k\,d\theta\,dk$$

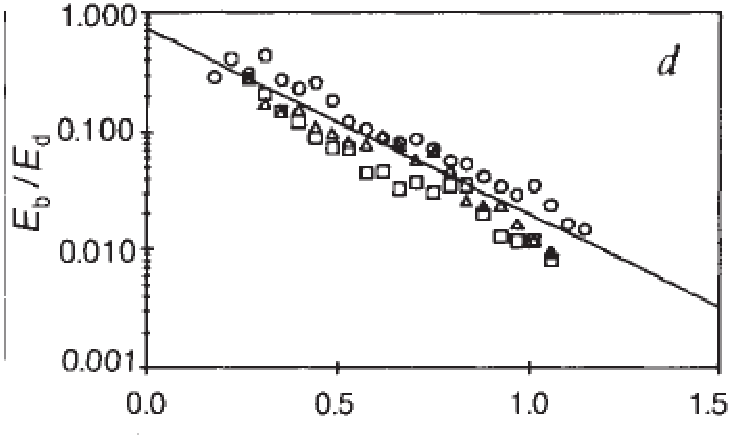
$$\Lambda(k)dk = \frac{\beta(k,\theta)k^4\cos(\theta)\Psi(k,\theta)}{\omega b}d\theta\,dk \sim \frac{[m]}{[m]}\left[\frac{1}{m}\right]$$

$$= \frac{m}{m^2}$$

Phillips, 1984

Volume of Air Entrained

- The ratio between work required to keep air entrained against buoyancy (Ed) and the energy dissipation (Ed) is linear (Figure).



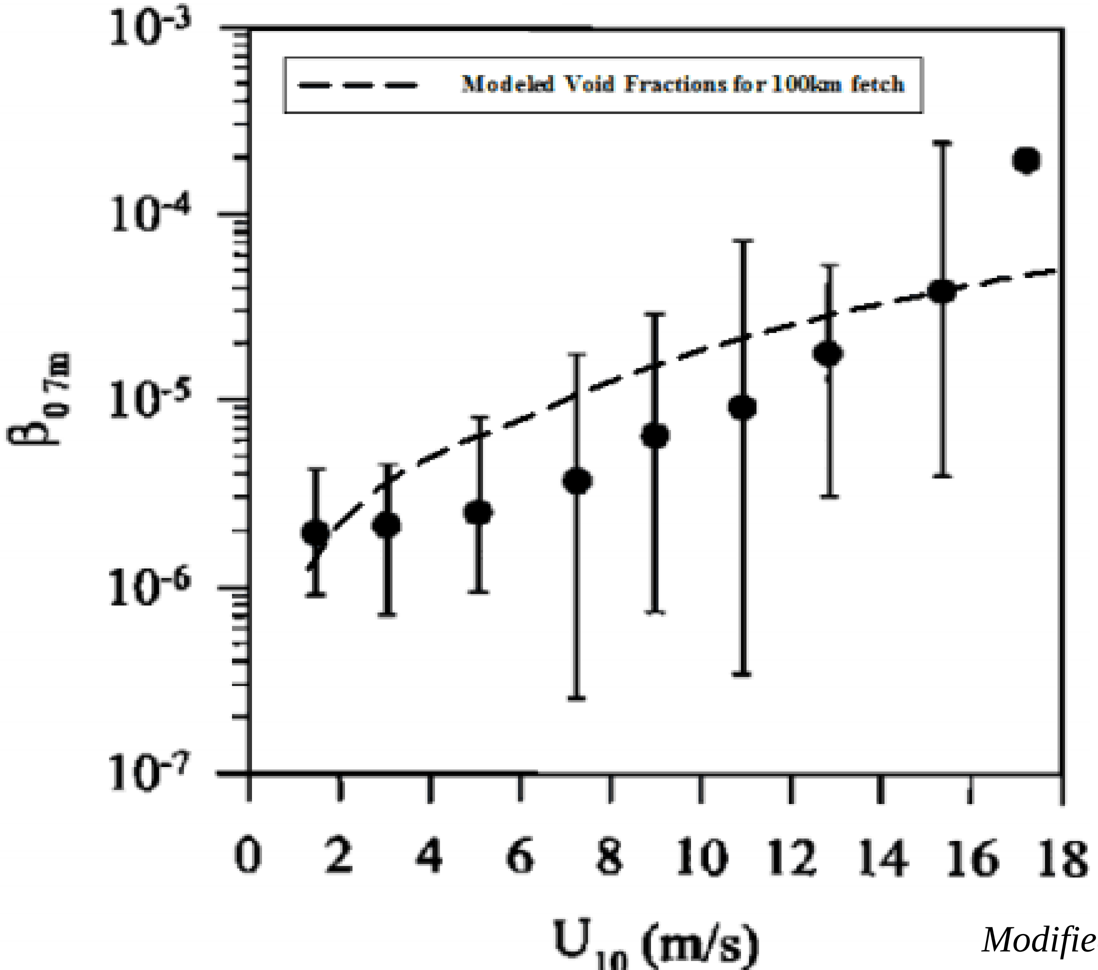
- The ration between the wave energy diss. To the max volume of air entrained is near constant.

$$V_a o = \frac{E(c)dc}{(10\%)g\lambda\rho} \sim \frac{[\frac{J}{m^2}]}{[\ ]\left[\frac{m}{s^2}\right][m][\frac{kg}{m^3}]} = \frac{[\frac{kgm^2}{m^2s^2}]}{[\ ]\left[\frac{m}{s^2}\right][m][\frac{kg}{m^3}]} = \left[\frac{m^3}{m^2}\right] \sim \frac{Vol}{unit\ area}$$

Lamarre and Melville, Nature (1991)

Modeled Void Fraction v. Obs

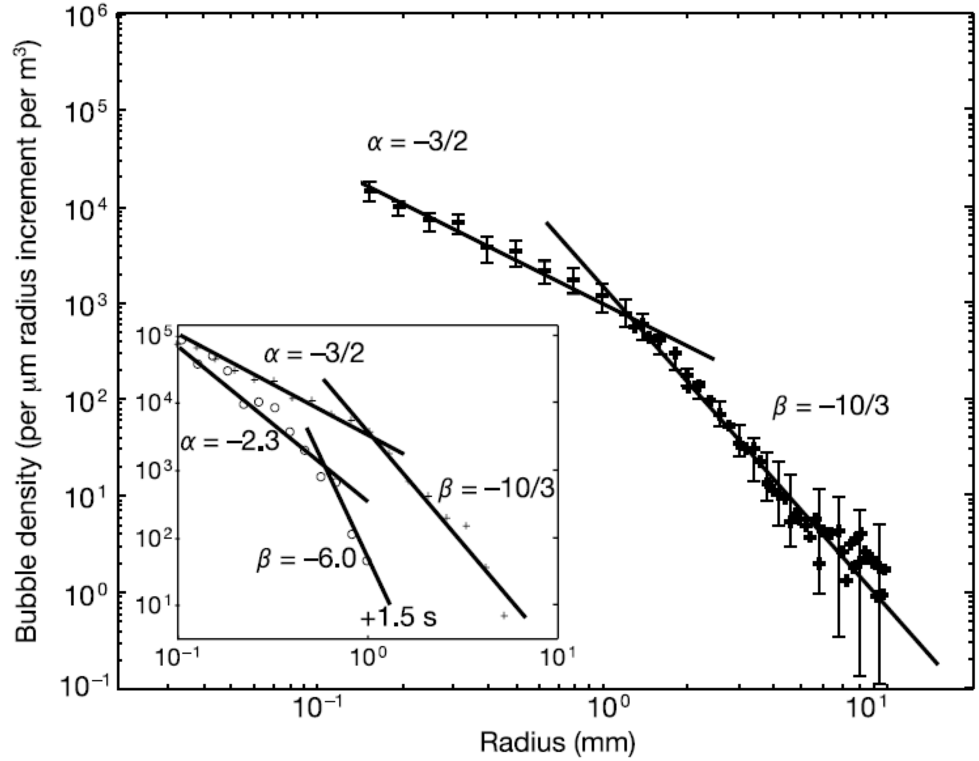
- Kalvoda et al. (2003) measured the breaking entrainment depth ~O(Hs). Dividing $V_{a}o$ by Hs gives the Void Fraction:



Modified from Terrill et al. (2001)

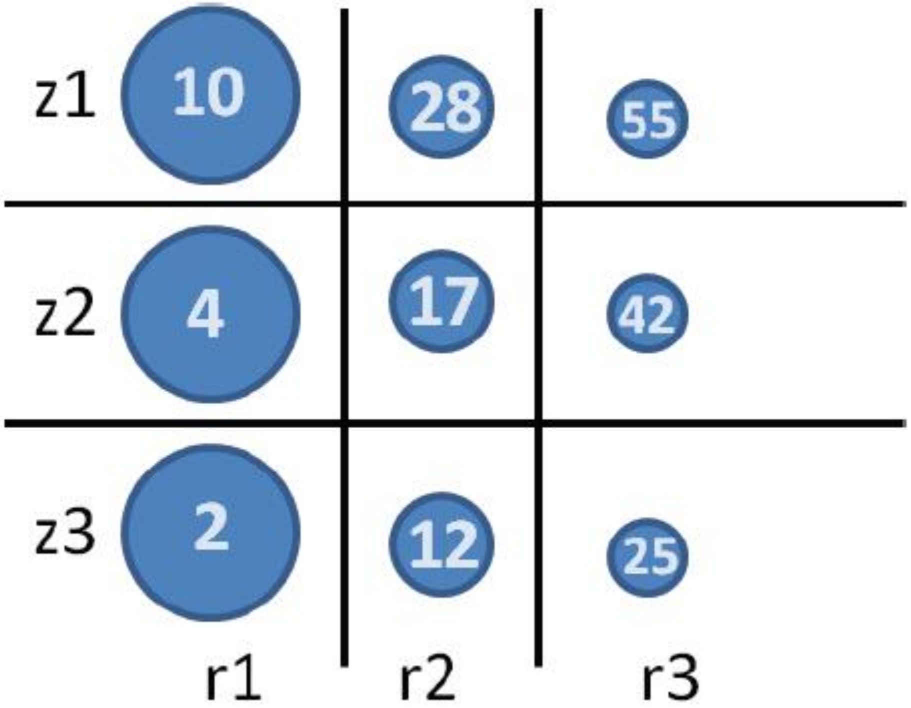
Future Work: Bubble Model

- Deane and Stokes, *Nature*, 2002 determined **bubble distributions** from the **volume of air entrained by a breaking wave**.



Proposed Bubble Model

- Characteristic Bubbles



	r1	r2	r3
z1	10	28	55
z2	4	17	42
z3	2	12	25

Proposed Velocity Field Beneath Breaking Waves

Mean and Turbulent Currents

- The velocity field below the ocean surface:

$$u = \overline{U} + U^{orb} + \tilde{u} + u'$$

- The mean, Eulerian component:

$$\begin{aligned} \overline{U} &= \overline{U_{stokes}} + \overline{U_{shear}} \\ &= \omega k a^2 e^{2k\zeta} + \frac{u_{*w}}{\kappa} \ln\left(\frac{\zeta}{z_0}\right) \end{aligned}$$

Wave Orbital Motion

- Wave Field from empirical wave spectrum by Elfouhaily et al. (1997):

$$\eta(x,t)=\sum_n a_n \cos(k_n x-\omega_n t+\phi_n)$$

- Wave Orbital Velocities from Mueller and Veron, 2009:

$$U_1^{orb}(x,\zeta,t)=\sum_n -a_n\omega_n\cos(k_n x-\omega_n t+\phi_n)\exp(-k_n\zeta)$$

$$U_3^{orb}(x,\zeta,t)=\sum_n a_n\omega_n\sin(k_n x-\omega_n t+\phi_n)\exp(-k_n\zeta).$$

Mueller and Veron (2009b)

Wave Breaking Eddy ($\tilde{u}$)

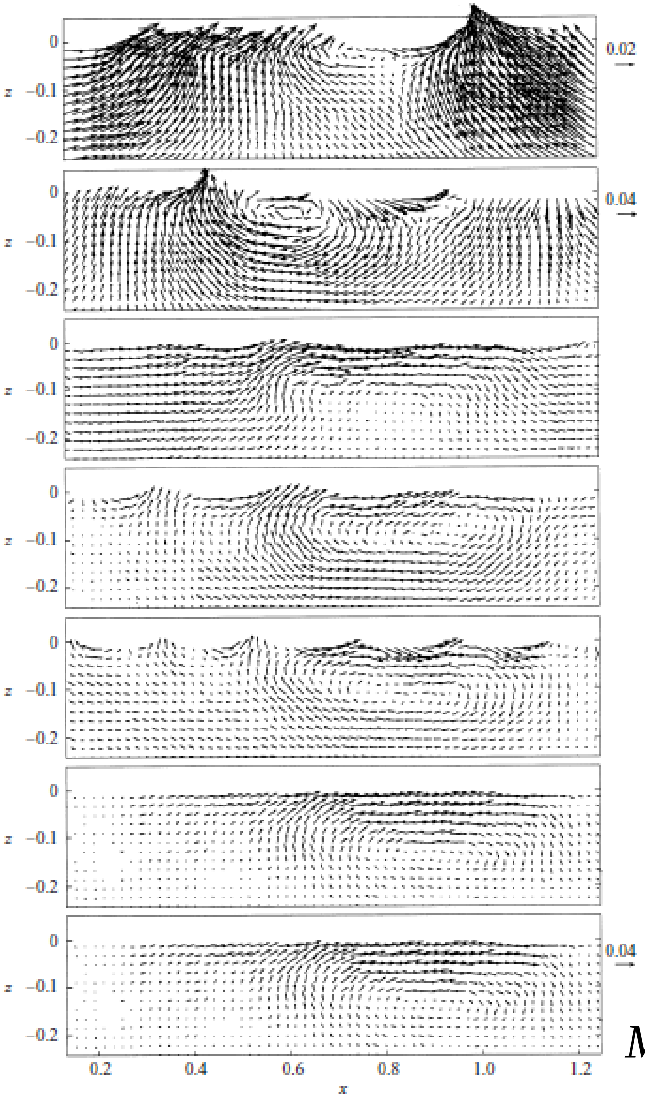


FIGURE 5. The mean velocity field at times $t = 3, 10.5, 26.5, 34.5, 42.5, 50$ and 58 for $S = 0.656$. For convenience in constructing the figure, the data points are decimated by a factor of 10 in each direction. The top panel shows large orbital velocities as the waves propagate across the measurement region to the right. Subsequently, the main feature that distinguishes the mean velocity is a coherent vortex slowly propagating to the right. Note the different scale for the first panel.

Melville et al. (2002)

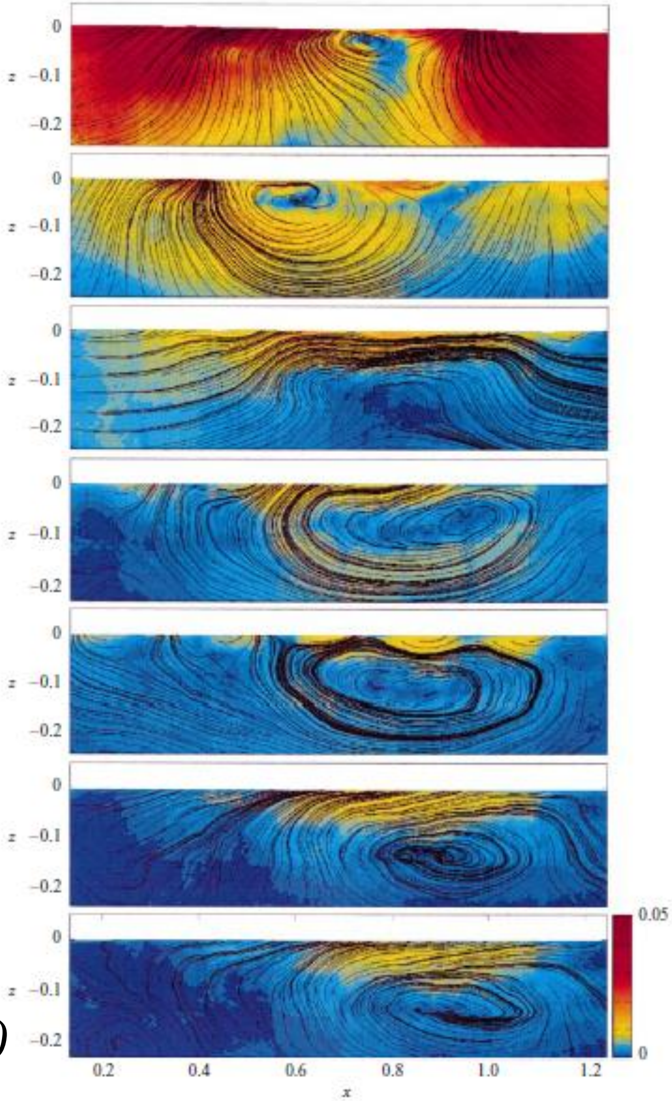
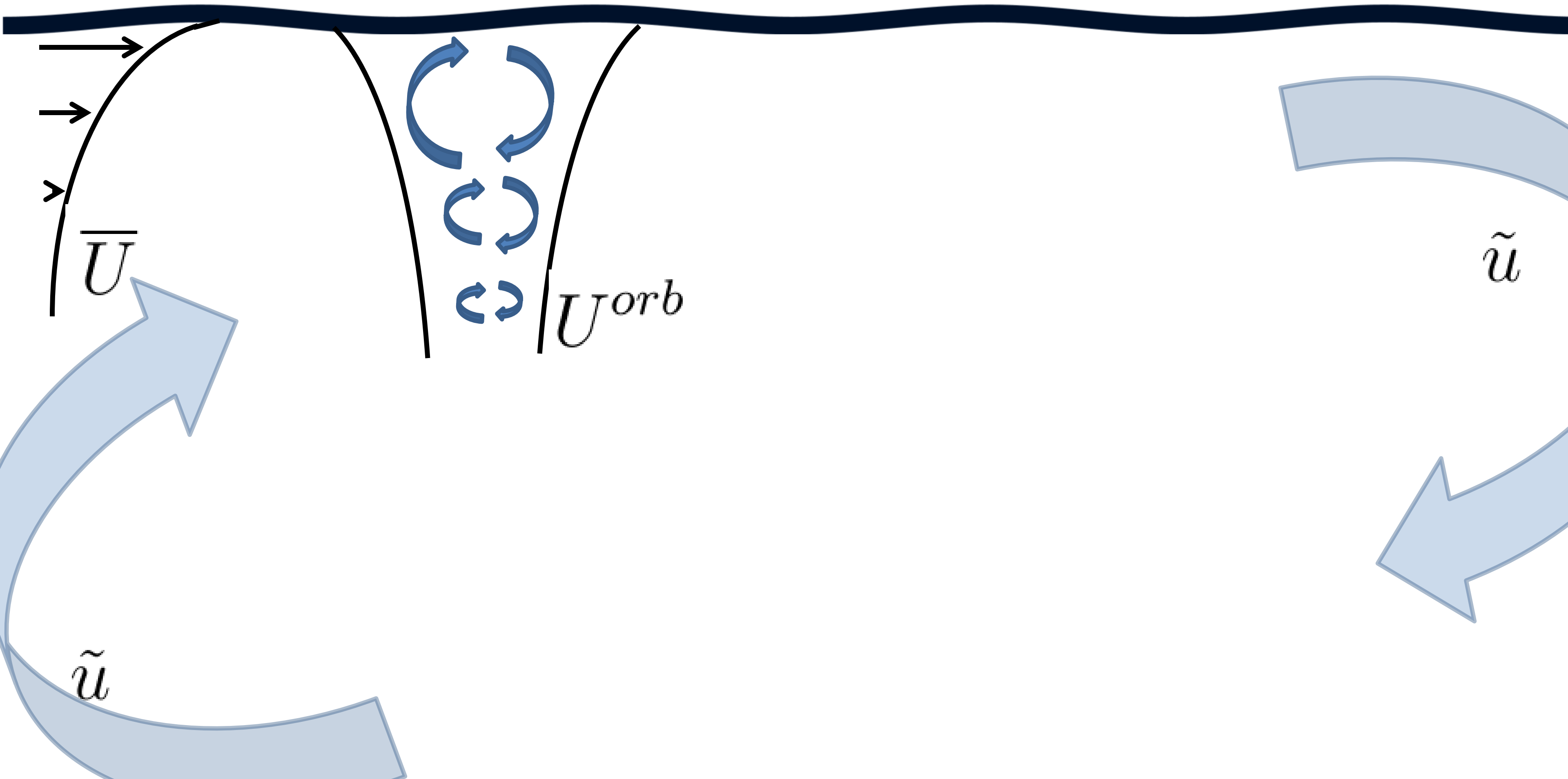
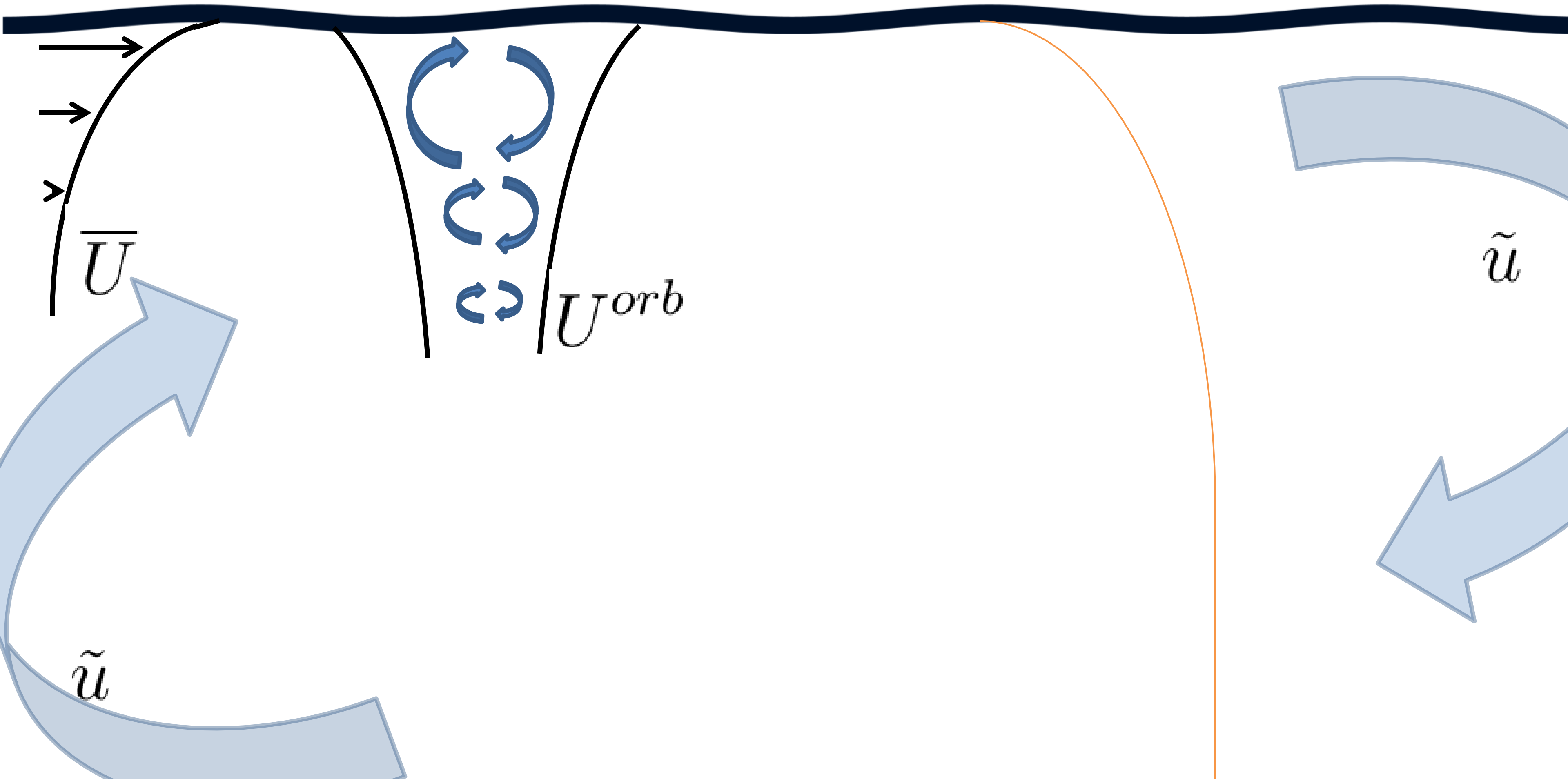


FIGURE 6. The streamlines corresponding to the mean flow shown in figure 5. The colour code shows the magnitude of the mean velocity. As shown in figure 5, the main feature is a coherent vortex which slowly propagates downstream and deepens.

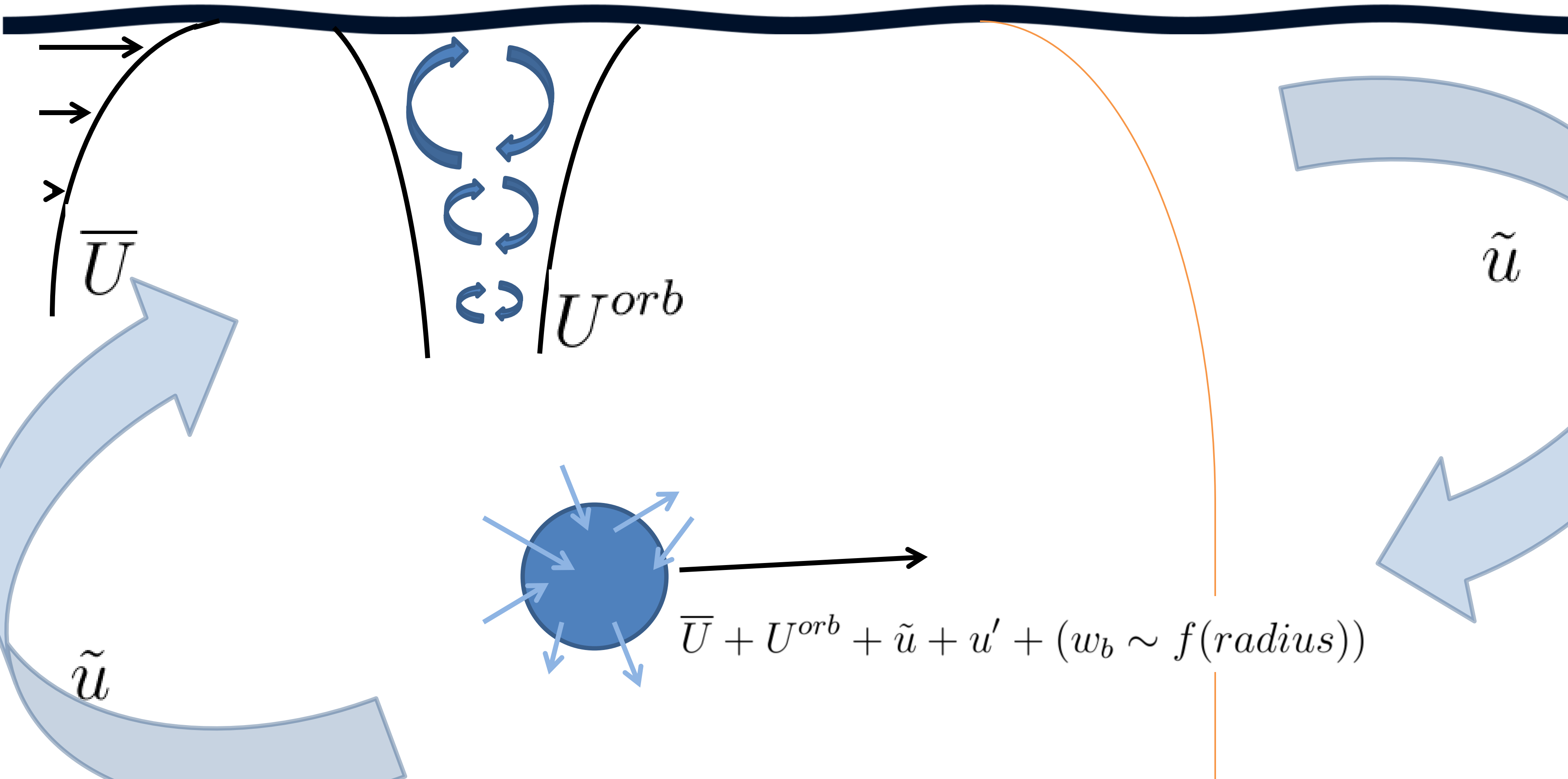
Proposed Bubble Model



Proposed Bubble Model



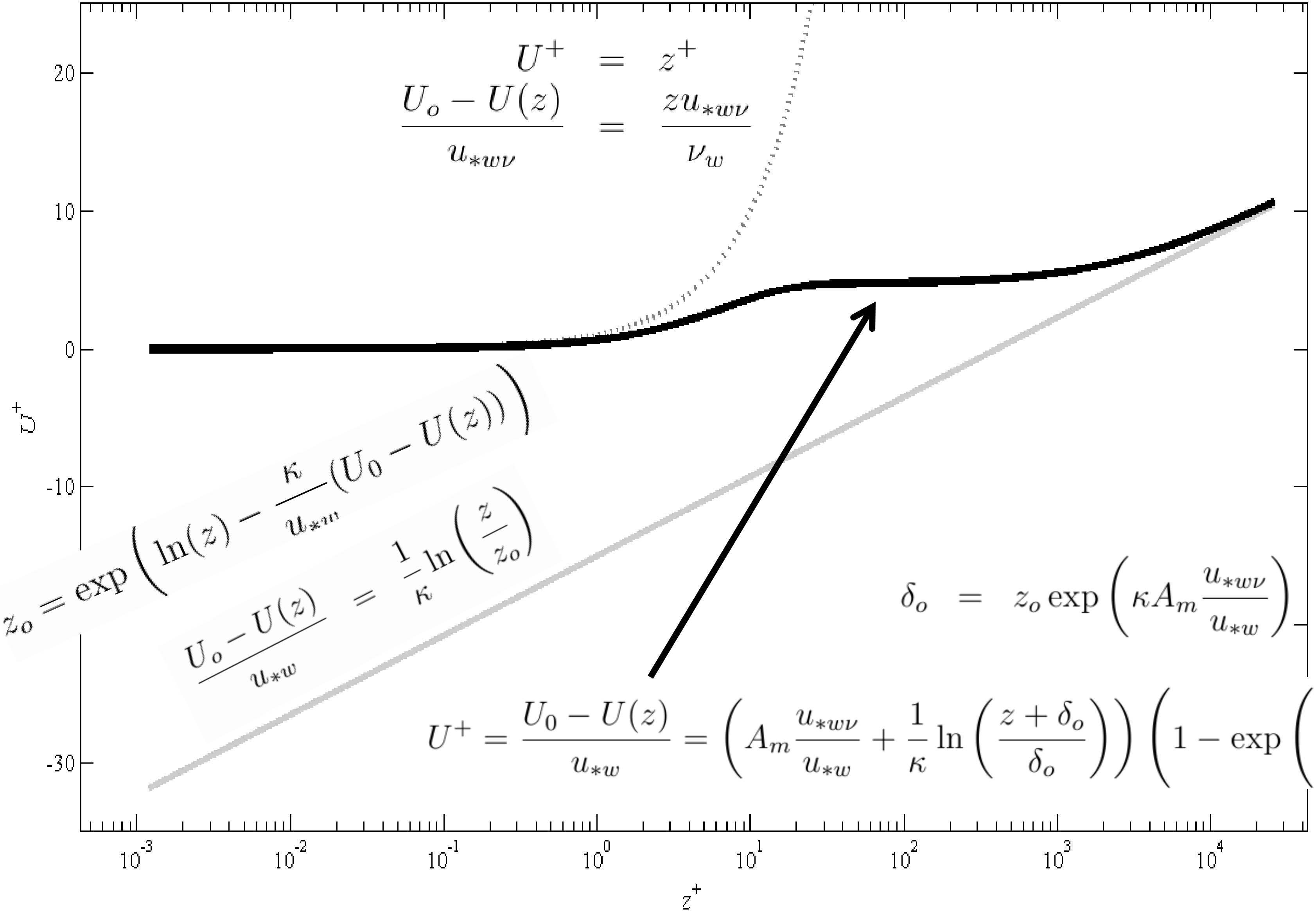
Proposed Bubble Model



Progress on Velocity Field Beneath Breaking Waves

Previous Drift Profile for Aerodynamically Rough Flow

Log-Linear Profiles in Water for U_{10} = 14



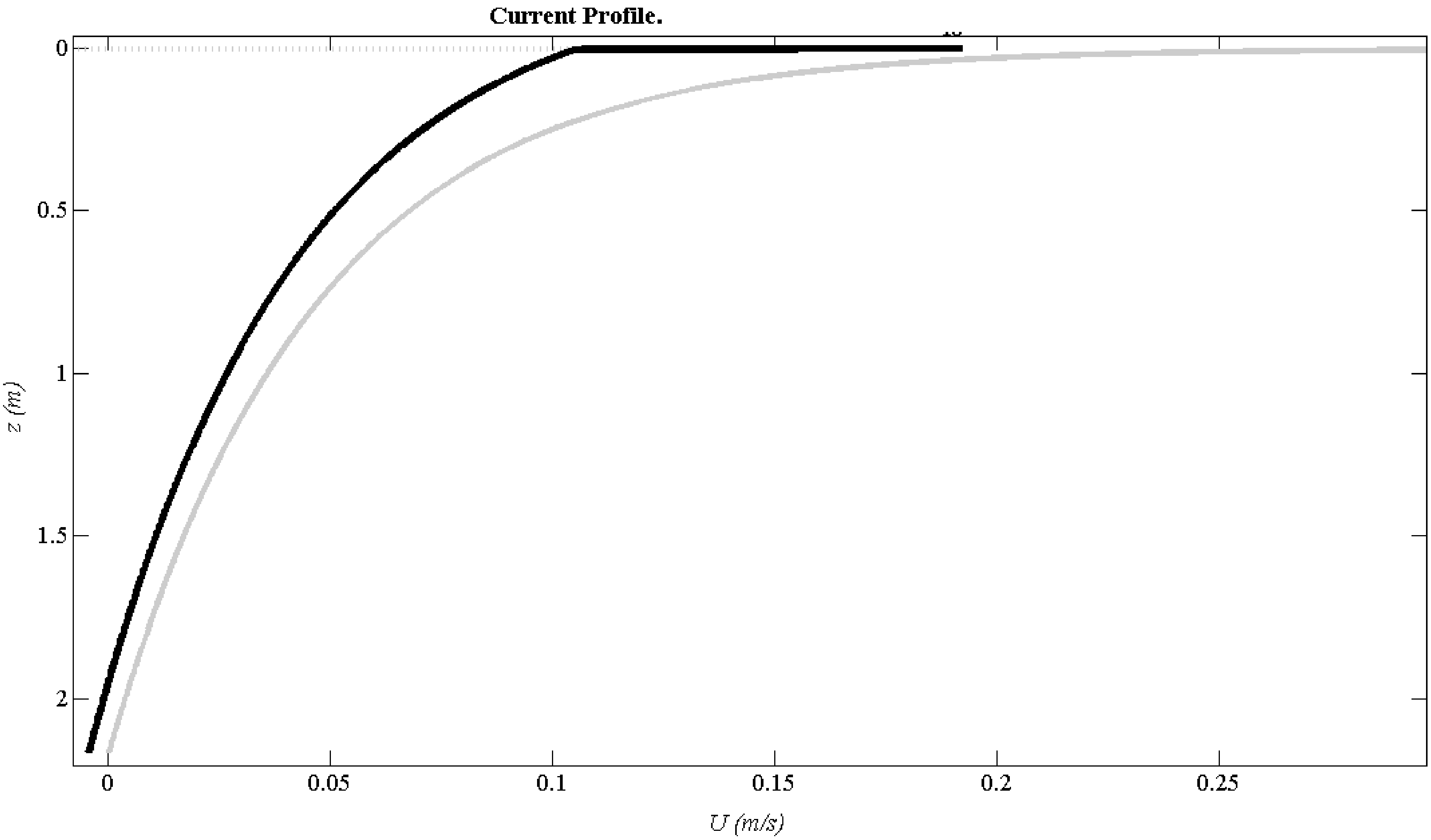
$$\begin{aligned} U^+ &= z^+ \\ \frac{U_o - U(z)}{u_{*w\nu}} &= \frac{z u_{*w\nu}}{\nu_w} \end{aligned}$$

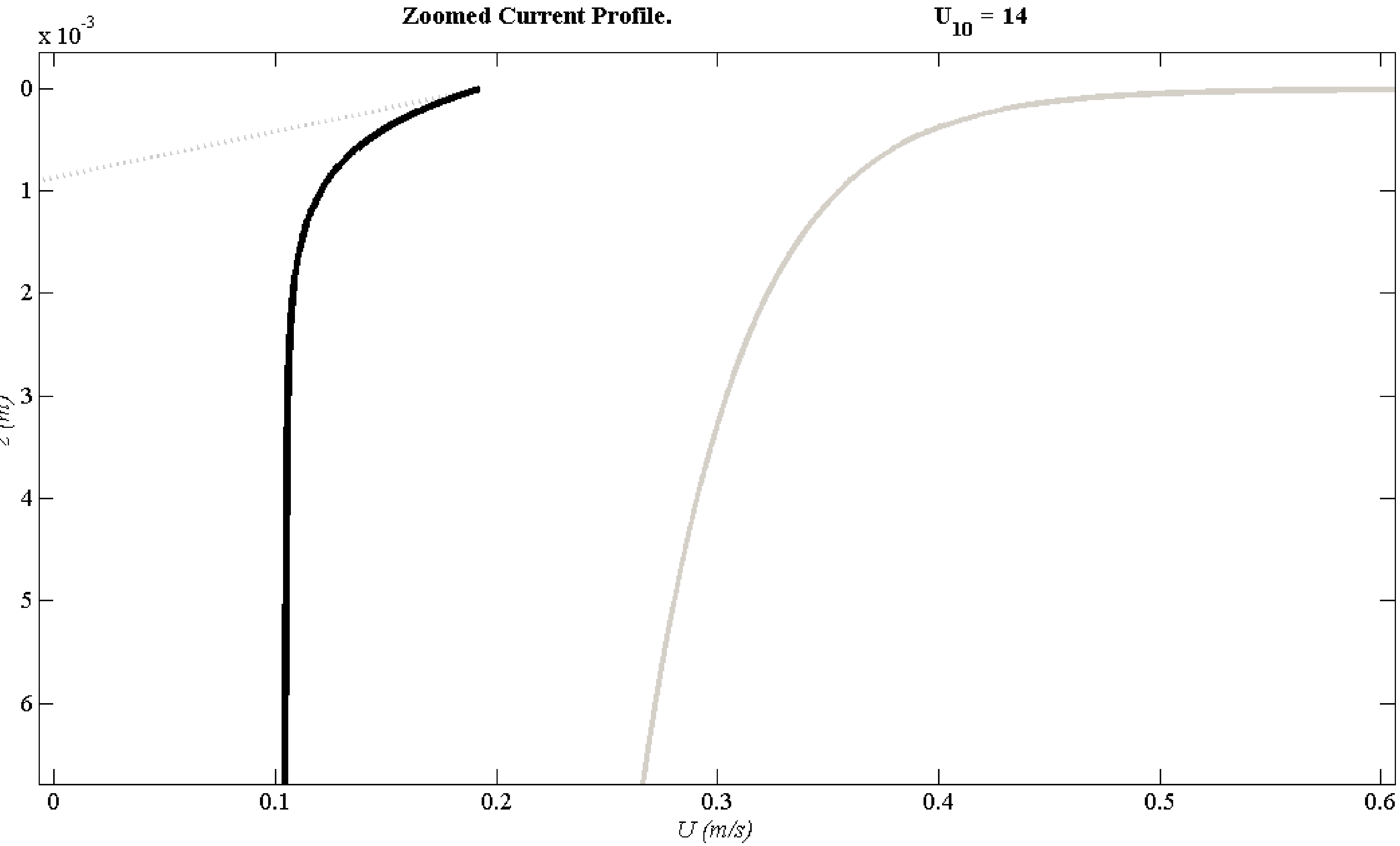
$$z_o = \exp\left(\ln(z) - \frac{\kappa}{u_{*w}}(U_0 - U(z))\right)$$

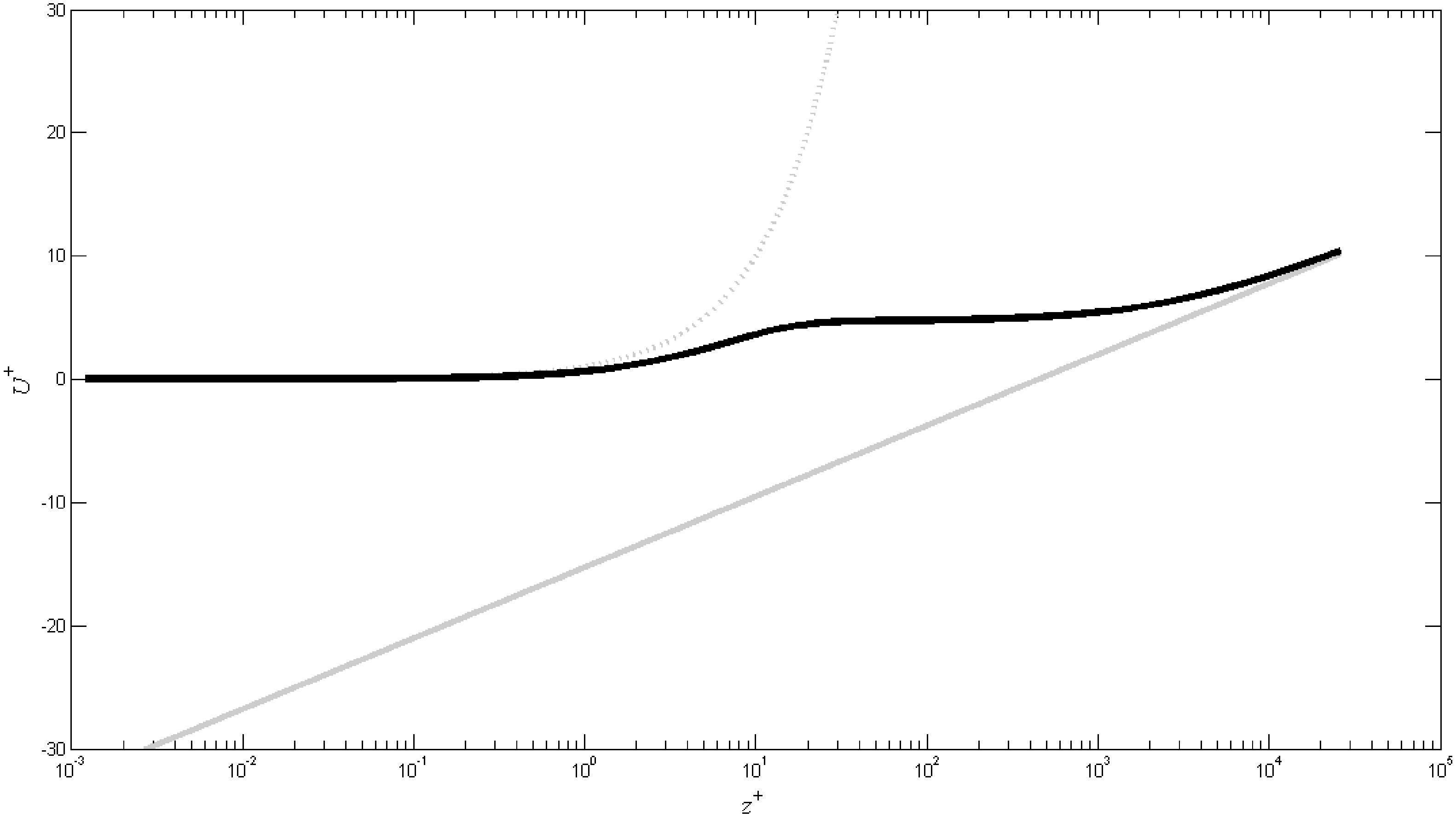
$$\frac{U_o - U(z)}{u_{*w}} = \frac{1}{\kappa}\ln\left(\frac{z}{z_o}\right)$$

$$\delta_o = z_o \exp\left(\kappa A_m \frac{u_{*w\nu}}{u_{*w}}\right)$$

$$U^+ = \frac{U_0 - U(z)}{u_{*w}} = \left(A_m \frac{u_{*w\nu}}{u_{*w}} + \frac{1}{\kappa}\ln\left(\frac{z+\delta_o}{\delta_o}\right)\right)\left(1 - \exp\left(\frac{-z^+}{A_m \frac{u_{*w\nu}}{u_{*w}}}\right)\right)$$

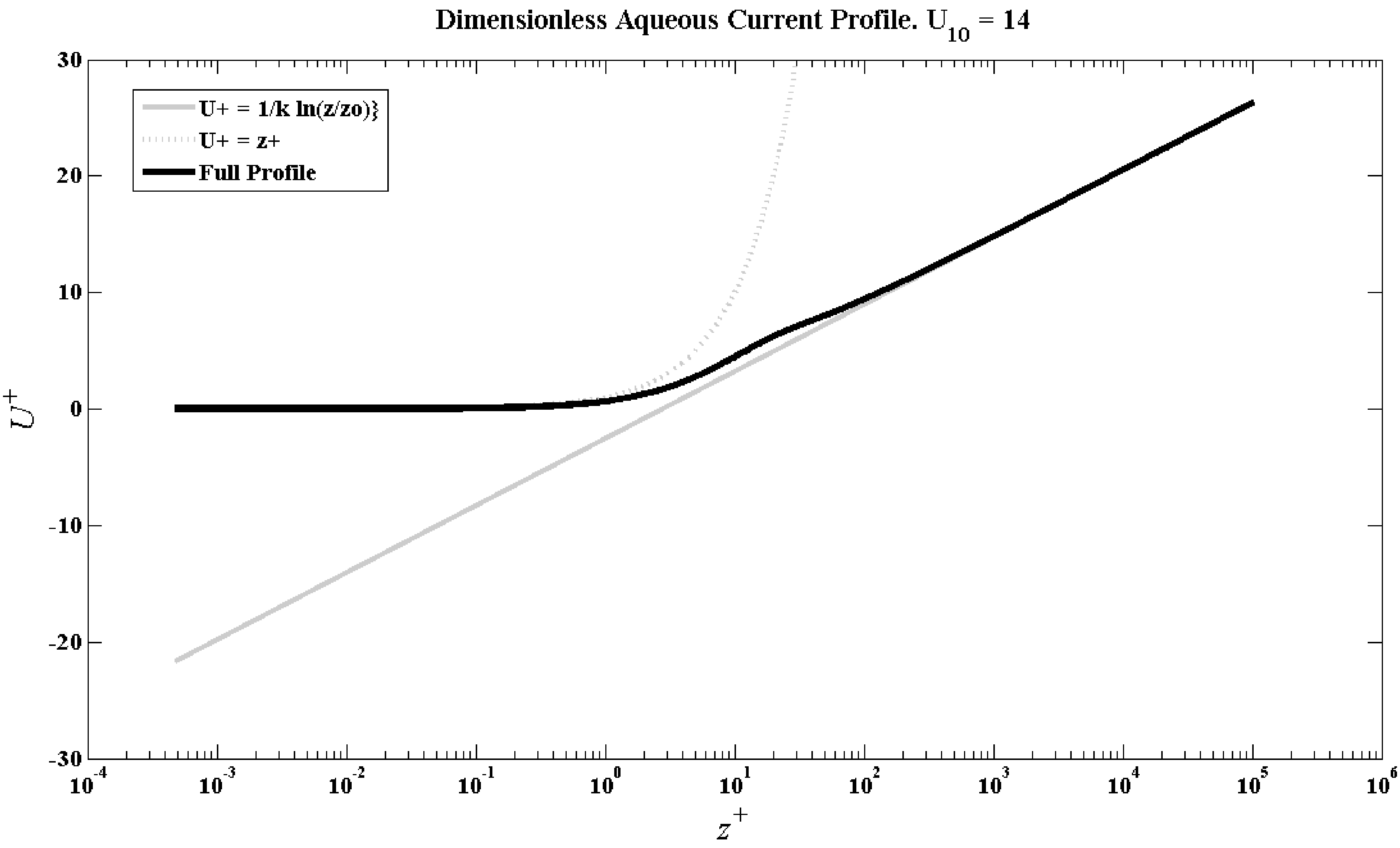

Current Profile.
0
0.5
1
1.5
2
z (m)
0
0.05
0.1
0.15
0.2
0.25
U (m/s)

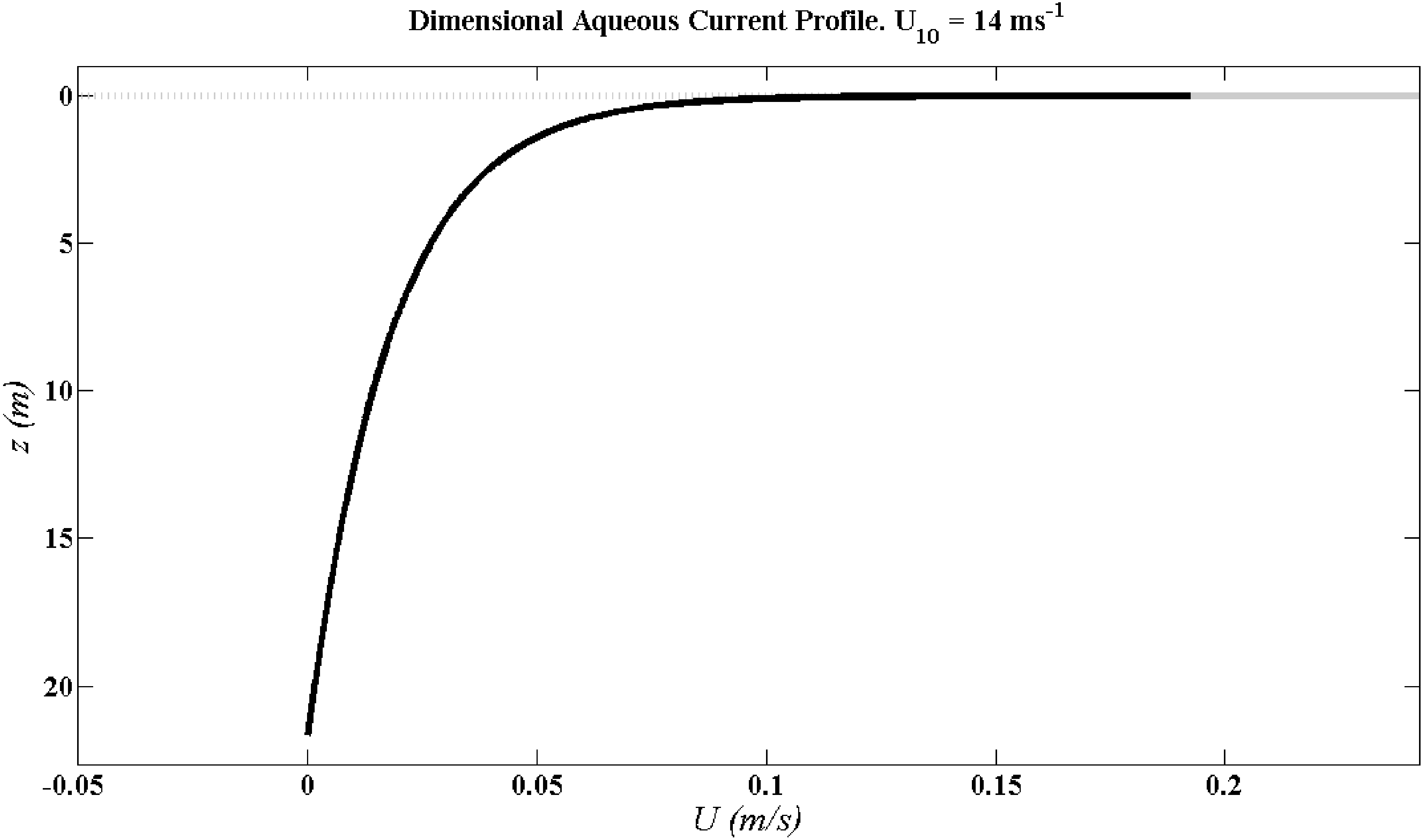

Zoomed Current Profile.
U_{10} = 14
x 10^{-3}
0
1
2
3
4
5
6
z (m)
0
0.1
0.2
0.3
0.4
0.5
0.6
U (m/s)


30
20
10
0
-10
-20
-30
U^+
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
z^+

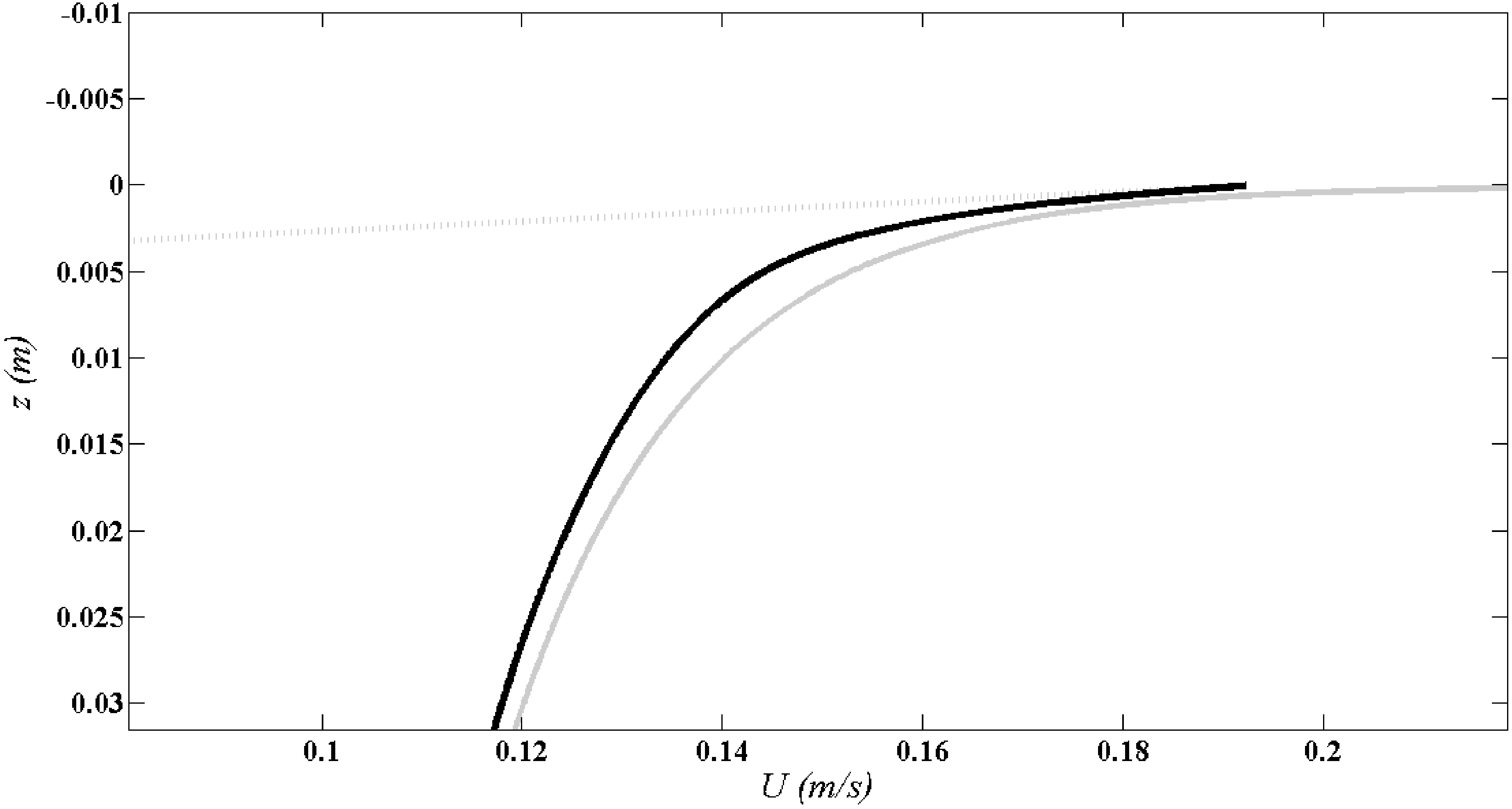
Viscous and Turbulent Layer Profiles Never Converged to Continuous Profile

Updated Drift Profile for Aerodynamically Rough Flow


Dimensionless Aqueous Current Profile. U10 = 14
U+ = 1/k ln(z/zo)}
U+ = z+
Full Profile
30
20
10
0
-10
-20
-30
U+
10-4
10-3
10-2
10-1
100
101
102
103
104
105
106
z+

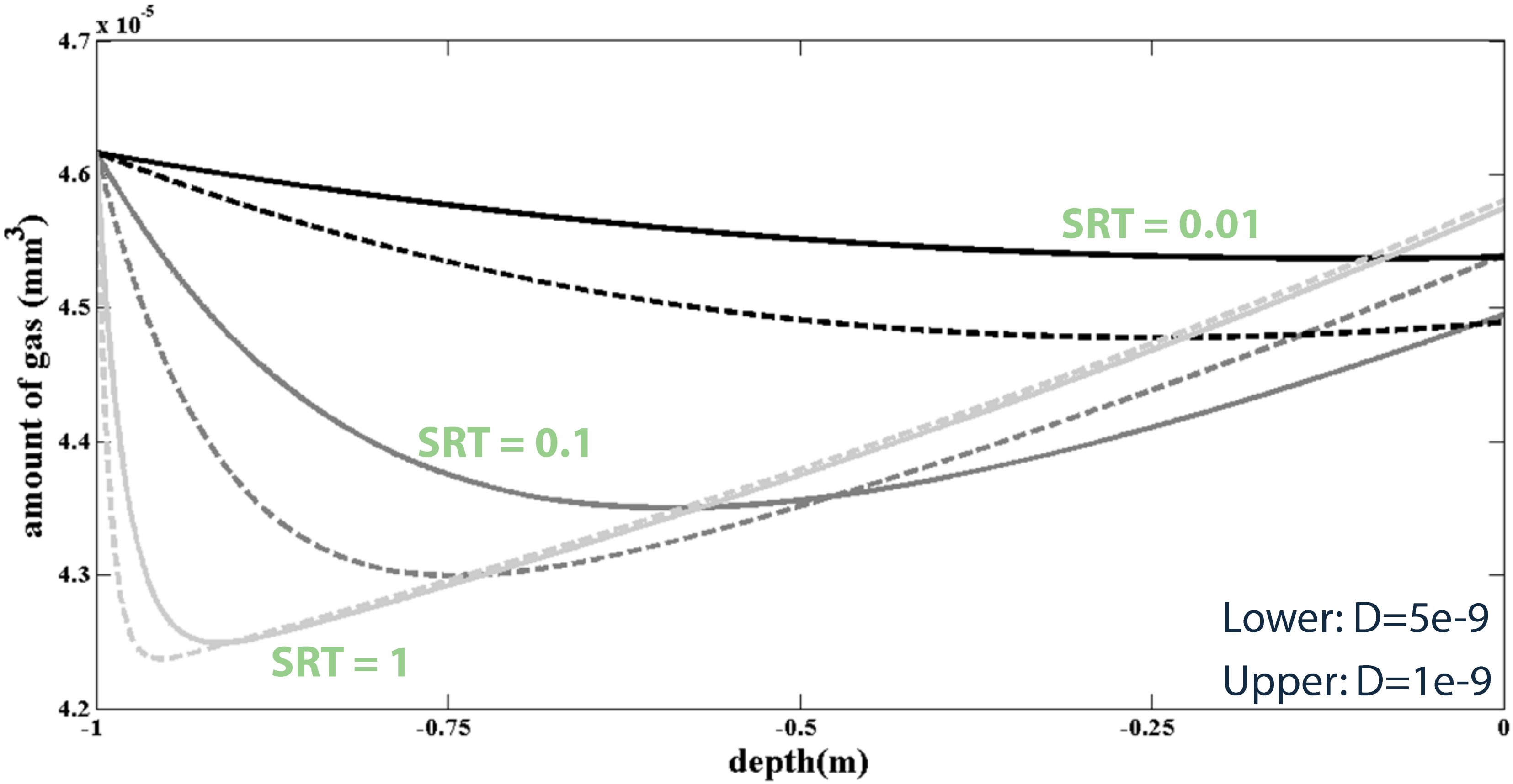

Dimensional Aqueous Current Profile. U_{10} = 14 ms^{-1}
0
5
10
15
20
z (m)
-0.05
0
0.05
0.1
0.15
0.2
U (m/s)

Zoomed in!


-0.01
-0.005
0
0.005
0.01
0.015
0.02
0.025
0.03
z (m)
0.1
0.12
0.14
0.16
0.18
0.2
U (m/s)

Older Bubble Model Progress

Single Bubble Flux v. Depth



SRT = 0.01

SRT = 0.1

SRT = 1

Lower: D=5e-9

Upper: D=1e-9

$$P_b = \frac{\sum(n_i)RT}{V} = P_{ao} + \rho g z + \frac{2\sigma}{r}$$

$$X_i = \frac{n_i}{\sum n_i}$$

$$\frac{dn_i}{dt} = k_i r^2 S_i (P_w X_{w_i} - P_b X_{b_i})$$

Wave Augmented Scalar Flux